Practice Problems 3 - Solutions

1 a) $\sum_{i=1}^{n}(4i) = 2n(n+1)$

Prove the basis case: $n=1$ $\quad \sum_{i=1}^{1}(4i) = 4\cdot 1 = 4$

$2\cdot 1\cdot(1+1) = 4$ ✓

Assume that the statement is true for $k \geq 1$.

Induction hypothesis: $\boxed{\sum_{i=1}^{k}(4i) = 2k(k+1)}$ We assume that this is true! We can (and will!) use this.

We want to show that follows that the statement is also true for $k+1$

Induction: $\sum_{i=1}^{k+1}(4i) = \sum_{i=1}^{k}(4i) + 4\cdot(k+1) \overset{\text{use induction hypothesis}}{=} 2k(k+1) + 4(k+1)$

$= (k+1)(2k+4) = 2(k+1)(k+2) = \boxed{2(k+1)((k+1)+1)}$ ∎

This is what we needed to show

b) $5^n - 1$ is divisible by 4

Prove the basis case $n=0$ $\quad 5^0 - 1 = 0$

↳ 0 is divisible by 4

(Alternatively you can use $n=1$)

Assume that the statement is true for $k \geq 0$.

Induction hypothesis: $5^k - 1$ is divisible by 4

$\Rightarrow \boxed{5^k - 1 = 4t}$ for some integer t

We want to show that follows that the statement is also true for $k+1$

Induction $\quad 5^{k+1} - 1 = 5\cdot 5^k - 1 = 5\cdot 5^k - 1 \underbrace{-4+4}_{0} = 5\cdot 5^k - 5 + 4$

$= 5(5^k - 1) + 4 \overset{\text{use induction hypothesis}}{=} 5\cdot 4t + 4 = 4\cdot(5t+1)$

↳ divisible by 4 ∎