HW #9

77.-

a) 0.288

b) 0.3643

c) 0.2778

80.-

a) −1.281

c) 1.645

f) 0

82.- $\mu = 15$ $\sigma = 3$

a) $P(X \le 17) = P\left(\frac{X-\mu}{\sigma} \le \frac{17-\mu}{\sigma}\right)$

$= P\left(Z \le \frac{17-15}{3}\right) = P(Z \le 0.377) = \underline{0.6469}$

b) $P(X \ge 11) = P\left(Z \ge \frac{11-15}{3}\right) = P(Z \ge -1.33) = \underline{0.908}$

84- $\mu = 8$ $\sigma = 2$

a.) $P(X \le X_0) = 0.2$

$P\left(Z \le \frac{X_0 - \mu}{\sigma}\right) = 0.2$

then find Z_0 such that

$P(Z \le Z_0) = 0.2$; $Z_0 = -0.841$

So $\frac{X_0 - \mu}{\sigma} = -0.841$

$\frac{X_0 - 8}{2} = -0.841$

$X_0 = (2)(-0.841) + 8$

$\boxed{X_0 = 6.31}$

b) $X_0 = 9.157$

c) $X_0 = 11.289$

85. Let X = amount spent

X is $N(600, 40)$

Find $P(X > 700) = P\left(\frac{X-\mu}{\sigma} > \frac{700-\mu}{\sigma}\right)$

$= P\left(Z > \frac{700-600}{40}\right)$

$= P(Z > 2.5) = 0.0062.$

86. Let X_0 refer to the amounted budget.

Then $P(X > X_0) = P\left(\frac{X-\mu}{\sigma} > \frac{X_0-\mu}{\sigma}\right) = 0.10$

$= P\left(Z > \frac{X_0-600}{40}\right) = 0.10$

$= P(Z > 1.28) = 0.10$

so $\frac{X_0-600}{40} = 1.28$

$X_0 = (40)(1.28) + 600 = 651.2$

$\boxed{X_0 = 651.2}$

87- Let X = diameter of Steel Shafts.

X is $N(1.005, 0.01)$.

a.) $P(0.98 \le X \le 1.02)$

$= P\left(\frac{0.98-\mu}{\sigma} \le Z \le \frac{1.02-\mu}{\sigma}\right)$

$= P\left(\frac{0.98-1.005}{0.01} \le Z \le \frac{1.02-1.005}{0.01}\right)$

$= P(-2.5 \le Z \le 1.5) = 0.926$

So P(fail to meet Specifications) $= 1-0.926 = \underline{0.074}$

b) Find μ Such that

$P\left(\frac{0.98-\mu}{0.01} \le Z \le \frac{1.02-\mu}{0.01}\right)$ is maximazed

$\mu = 1$, Since with $\mu = 1$, we obtain

$P(-2 \le Z \le 2)$ which is maximal by

the Symmetry and Shape of the $N(0,1)$.

89 - Let X = resistance of wires

X is $N(0.13, 0.005)$

a) $P(0.12 < X < 0.14) = P\left(\frac{0.12-0.13}{0.005} < Z < \frac{0.14-0.13}{0.005}\right)$

$= P(-2 < Z < 2) = 0.95$

b) Let W = # wires to meet spec.

W is Binomial $(4, 0.95)$

$P(W=4) = (0.95)^4 = 0.83$

91. Let X = thermistor resistance

X is $N(10\,000, 4000)$

$$P(8000 < X < 15000) = P\left(\frac{8000-10000}{4000} < Z < \frac{15000-10000}{4000}\right)$$

$$= P(-0.5 < Z < 1.25) = 0.5859$$

138. X is uniform (a,b)

$$f_X(x) = \begin{cases} \frac{1}{b-a} & \text{if } x \in (a,b) \\ 0 & x \notin (a,b) \end{cases}$$

a)

$$M_X(t) = E\, e^{tY} = \int_{-\infty}^{\infty} e^{tX} f(x)dx = \int_a^b \frac{e^{tx}}{b-a} f(x) = \frac{1}{t(b-a)}\left[e^{tx}\right]_a^b = \frac{e^{bt} - e^{at}}{t(b-a)}.$$

So for $Y = cX + d$

$$M_Y(t) = e^{td} M_X(ct) = e^{td}\left(\frac{e^{bct} - e^{act}}{ct(b-a)}\right) = \frac{e^{t(bc+d)} - e^{t(ac+d)}}{t(bc-ac)}$$

b)

Y is uniform on $(ac+d, bc+d)$.

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$$M(t) = E e^{tx} = \int_0^\infty e^{tx} \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}} \, dx$$

$$= \int_0^\infty \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x\left(\frac{1}{\beta}-t\right)} \, dx$$

$$= \frac{1}{(1-\beta t)^\alpha} \underbrace{\int_0^\infty \frac{1}{\Gamma(\alpha)\left(\frac{\beta}{1-\beta t}\right)^\alpha} x^{\alpha-1} e^{-x\left(\frac{1-\beta t}{\beta}\right)} \, dx}_{\text{this integrates to } 1.}$$

$$= (1-\beta t)^{-\alpha}$$

7.1 Let $X = \#$ of female applicants among the final three

a. $P(X=x) = \begin{cases} \dfrac{\binom{6}{x}\binom{4}{3-x}}{\binom{10}{3}} & x = 0, 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$

b. $Y = 3 - X$, so $X = 3 - Y$

c.

$$P(Y=y) = \begin{cases} \dfrac{\binom{6}{3-y}\binom{4}{y}}{\binom{10}{3}} & y = 0, 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

7.3 Similar to 7.1

7.4 a) $C = 300 + 40(Y-4) = 140 + 40Y$

b) $F_C(c) = P(C \le c) = P(140 + 40Y \le c)$

$$= P\left(Y \le \frac{c-140}{40}\right) = F_Y\left(\frac{c-140}{40}\right)$$

c. from 7.3 X is negative binomial

$$P(X \le x) = \binom{x+4-1}{4-1}(1-p)^4 p^x$$

$$C = 300 + 40(X+4) = 460 + 40X$$

d

$$F_C(c) = P(C \le c) = P(460 + 40X \le c)$$

$$= P\left(X \le \frac{c-460}{40}\right) = F_X\left(\frac{c-460}{40}\right)$$

7.5

a. $F_{Y_1}(y) = P(Y_1 \le y) = P(2X - 1 \le y) = P\left(X \le \frac{y+1}{2}\right)$

$$= F_X\left(\frac{y+1}{2}\right) = \int_0^{\frac{y+1}{2}} 4(1-2x)dx = [4x - 4x^2]\Big|_0^{\frac{y+1}{2}} = \left[2y + 2 - 4\frac{(y+1)^2}{4}\right] = y^2 + 4y + 3$$

$$F_{Y_1}(y) = \frac{dF_{Y_1}(y)}{dy} = \begin{cases} \frac{d(y^2+4y+3)}{dy} = 2(y+2) & -1 \le y \le 0 \\ 0 & otherwise \end{cases}$$

b. $F_{Y_2}(y) = P(Y_2 \le y) = P(1 - 2X \le y) = P\left(X \ge \frac{1-y}{2}\right)$

$$= 1 - F_X\left(\frac{1-y}{2}\right) = 1 - \int_0^{\frac{1-y}{2}} 4(1-2x)dx = 1 - [4x - 4x^2]\Big|_0^{\frac{1-y}{2}} = 1 - \left[2 - 2y - (1 - 2y + y^2)\right]$$

$= 2 - y^2$, where $0 \le \frac{1-y}{2} \le 0.5 \rightarrow 0 \le y \le 1$

$$F_{Y_2}(y) = \frac{dF_{Y_2}(y)}{dy} = \begin{cases} \frac{d(2-y^2)}{dy} = -2y & 0 \le y \le 1 \\ 0 & otherwise \end{cases}$$

c. $F_{Y_3}(y) = P(Y_3 \le y) = P(X^2 \le y) = P(X \le \sqrt{y})$

$$= F_x(\sqrt{y}) = \int_0^{\sqrt{y}} 4(1-2x)dx = \left[4x - 4x^2\right]\Big|_0^{\sqrt{y}} = 4\sqrt{y} - 4y$$

$0 \le \sqrt{y} \le 0.5 \rightarrow 0 \le y \le 0.25$

$$f_{Y_3}(y) = \begin{cases} \frac{d(4\sqrt{y} - 4y)}{dy} = \frac{2}{\sqrt{y}} - 4 & 0 \le y \le 0.25 \\ 0 & otherwise \end{cases}$$

7.7

a. $F_Y(y) = P(Y \le y) = P(10X - 4 \le y) = \left(X \le \frac{y+4}{10} \right) = F_X\left(\frac{y+4}{10} \right)$

$$= \begin{cases} \int_0^{\frac{y+4}{10}} x\,dx = \frac{(y+4)^2}{200} & -4 \le y \le 6 \\ F_x(1) + \int_1^{\frac{(y+4)}{10}} 1\,dx = \frac{1}{2} + \frac{y+4}{10} - 1 = \frac{y-1}{10} & 6 \le y \le 11 \end{cases}$$

$$F_Y(y) = \frac{df_Y(y)}{dy} = \begin{cases} \frac{d}{dy}\frac{(y+4)^2}{200} = \frac{y+4}{100} & -4 \le y \le 6 \\ \frac{d}{dy}\frac{y-1}{10} = \frac{1}{10} & 6 \le y \le 11 \\ 0 & otherwise \end{cases}$$

b. $E(Y) = \int_{-4}^{6} y\frac{(4+y)}{10}dy + \int_6^{11} \frac{y}{10}dy = \frac{1}{100}\left(\frac{y^3}{3} + 2y^2 \right)\Bigg|_{-4}^{6} + \frac{y^2}{20}\Bigg|_6^{11}$

$\frac{1}{100}\left(\frac{18}{3} + 72 + \frac{64}{3} - 32 \right) + \frac{1}{20}(11^2 - 36) = \frac{4}{3} + \frac{17}{4} = \frac{67}{12} = 5.583$

c. $E(X) = \int_0^1 x^2dx + \int_1^{1.5} x\,dx = \frac{1}{3} + \frac{1}{2}\left(\frac{9}{4} - 1 \right) = \frac{1}{3} + \frac{9}{8} - \frac{1}{2} = \frac{23}{24} = 0.9583$

$E(Y) = E(10X - 4) = 10E(X) - 4 = \frac{230}{24} - 4 = \frac{67}{12} = 5.583$