5.3

a. $P(-1<X<1)=\int_{-1}^{1}\frac{x^2}{3}dx=0.222$

b. $P(1<X<3)=\int_{1}^{2}\frac{x^2}{3}dx=0.778$

c. $P(X\leq 1 \mid X\leq 1.5)=\dfrac{P(\{X\leq 1\}\cap\{X\leq 1.5\})}{P(X\leq 1.5)}$

$$=\frac{P(X\leq 1)}{P(X\leq 1.5)}=\frac{\int_{-1}^{1}\frac{x^2}{3}dx}{\int_{-1}^{1.5}\frac{x^2}{3}dx}=0.457$$

d.

$$F(x)=\begin{cases}0, & x<-1\\ \frac{x^3}{9}+\frac{1}{9}, & -1\leq x<2\\ 1, & x\geq 2\end{cases}$$

5.4

a. $c = 6$

b. $F(x) = \int_{-\infty}^{x} 6t(1-t)\,dt = \int_{0}^{x} 6t(1-t)\,dt = 6\frac{t^2}{2} - \frac{6t^3}{3}\Big|_0^x = 3t^2 - 2t^3\Big|_0^x$

$= 3x^2 - 2x^3 = -2x^3 + 3x^2$

just graph this polynomial.

c. $P(X > 0.75) = 1 - P(X \le 0.75) = 1 - [-2(0.75)^3 + 3(0.75)^2]$

$= 0.1562$

d. $P(X > 0.75 \mid X > 0.50) = \frac{P(X > 0.75)}{P(X > 0.5)} = \frac{0.1562}{0.5} = 0.3124$

5.7

a.

b. $P(0.25 < X < 0.75) = F(0.75) - F(0.25) = 0.1354$

c.

$$f(x) = \begin{cases} 0, & x < 0 \text{ or } x > 2 \\ \frac{3x^2}{2}, & 0 \le x \le 1 \\ \frac{1}{2}, & 1 \le x \le 2 \end{cases}$$

d.

5.15

a. $EX = 0.5$

$EX^2 = 0.3$

$V(X) = 0.05$

b. From $P(|X-\mu| < k\sigma) \geq 1 - \frac{1}{k^2}$

$1 - \frac{1}{k^2} = 0.7$, then $k = 2$

so

$P(|X-\mu| < 2\sigma) \geq 0.75$

$\Rightarrow P(-2\sigma < X-\mu < 2\sigma) \geq 0.75$

$\Rightarrow P(\mu - 2\sigma < X < \mu + 2\sigma) \geq 0.75$

the interval is $(\mu - 2\sigma, \mu + 2\sigma) = (0.5 - 2(\sqrt{0.05}), 0.5 + 2(\sqrt{0.05}))$

5.22 We need to find the p.d.f for X.

$$f(x) = \begin{cases} 0 , & x < 0 \text{ or } x > 10 \\ \frac{x}{80} , & 0 \le x \le 4 \\ \frac{1}{3} - \frac{x}{30} , & 4 \le x \le 10 \end{cases}$$

then

$$EX = \int_{-\infty}^{\infty} x f(x) dx = \int_{0}^{4} \frac{x^2}{80} dx + \int_{4}^{10} \frac{x}{3} - \frac{x^2}{30} dx .$$

5.28

Let X = amount of time that Henry is late.

X is Uniform on $(0, 20)$

So $f(x) = \begin{cases} \frac{1}{20} & , 0 \leq x \leq 20 \\ 0 & , \text{otherwise} \end{cases}$

a. $P(X > 15) = \int_{15}^{20} \frac{1}{20} dx = 0.25$

b. $EX = \frac{b+a}{2} = \frac{20+0}{2} = 10$

$V(x) = \frac{(b-a)^2}{12} = \frac{(20)^2}{12}$, so

$SD(x) = \sqrt{\frac{(20)^2}{12}} = \frac{20}{\sqrt{12}} = \underline{5.78}$

5.32

Let X = hour of day when defective blue ray was produced.

X is uniform on $(0, 8)$.

So $f(x) = \begin{cases} \frac{1}{8}, & \text{if } 0 \le x \le 8 \\ 0, & \text{otherwise} \end{cases}$

a. $P(X < 1) = \int_0^1 \frac{1}{8}\,dx = \frac{1}{8}$.

b. $P(X > 7) = \int_7^8 \frac{1}{8}\,dx = \frac{1}{8}$

c. $P(4 < X \le 5 \mid X > 4) = \frac{P(4 < X \le 5)}{P(X > 4)} = \frac{1}{4}$.

5.40

Let X = Cycle time for the trucks

X is uniform on $(50, 70)$

So, $f(x) = \begin{cases} \frac{1}{20}, & \text{if } 50 \le x \le 70 \\ 0, & \text{otherwise} \end{cases}$

a. $E X = \frac{70+50}{2} = 60$

$V(X) = \frac{(70-50)^2}{12} = 33.33$

b. Let N = # of trucks need, so

$N = \frac{E(X)}{15} = \frac{60}{15} = 4$

5.41 Let X = magnitude of earthquakes

X is $\exp(2.4)$

a.

$P(X \le 2.5) = F(2.5) = 1 - e^{-2.5/2.4} = 0.647$

b.

$P(X > 4) = 1 - F(4) = e^{-4/2.4} = 0.189$

c. $P(2 < X < 3) = F(3) - F(2) = e^{-2/2.4} - e^{-3/2.4} = 0.148.$

5.42

Let X = # of earthquakes in the next ten earthquakes to strike that will exceed 5.0.

Let $p = P(\text{earthquake exceeds } 5.0) = \underline{e^{-5/2.4}}$.

So X is binomial $(10, p)$

$$P(X \geq 1) = 1 - P(X=0) = 1 - \binom{10}{0} p^0 (1-p)^{10} = 1 - (1 - e^{-5/2.4})^{10}$$

5.45 Let X = waiting time between successive customer arrivals.

$E X = 0.5$
$V(X) = (0.5)^2 = 0.25$

5.46 Let X = distances between randomly selected trees.

X is exponential with $\theta = 40$

a. $P(X > 30) = 1 - F(30) = 0.472$

b. $P(X > 80 \mid X > 50) = P(X > 30) = 1 - F(30)$.

by memoryless property

c. $P(X > d) = 1 - F(d) = 1 - (1 - e^{-\frac{d}{40}}) = e^{-\frac{d}{40}} \geq 0.50$

solving for d, $\frac{-d}{40} \geq \ln(0.50) \Rightarrow d = -40\ln(0.50) = \underline{27.7}$

5.60

Let A = Cindy will be the last of the three customers to leave

Let X = Service time for customer 1

Y = Service time for customer 2

Z = Service time for Cindy

X, Y, Z are exponentially distributed with mean 2.

So,

$P(A) = P($ Cindy is the last one to leave | one customer left$)$

by the memory less property, the previous equals to

$p($Service time for Cindy is greater than Service time for the other customer$) = P(X > t_1 \mid X > t_2) = P(X > t_1 - t_2)$

for $t_1 > t_2$

as we did in class on Tuesday

5.65

a. $E X = \alpha\beta = (0.4)(20) = 8$

$V(X) = \alpha\beta^2 = (0.4)(20)^2 = 160$

$\sigma = 12.6$

b. $P(|X-\mu| < k\sigma) \geq 1 - \frac{1}{k^2}$

$1 - \frac{1}{k^2} = 0.75 \quad \Rightarrow \quad k = 2$

so

$P(|X - 8| < (2)(12.6)) \geq 0.75$

$\Rightarrow \quad P(-25.2 < X - 8 < 25.2) \geq 0.75$

$P(17.2 < X < 33.2) \geq 0.75$

If X is gamma (α, β), the K^{th}

moment can be found:

$$EX^{K} = \int_0^\infty \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha+k-1} e^{-x/\beta} dx = \frac{\beta^k \Gamma(\alpha+k)}{\Gamma(\alpha)} \int_0^\infty \frac{x^{\alpha+k-1}}{\Gamma(\alpha+\beta)\beta^{\alpha+k}} e^{-x/\beta} d$$

$$= \beta^K \frac{\Gamma(\alpha+k)}{\Gamma(\alpha)}$$

this is useful for the next

Problem, 5.67

5.67

$E(L) = 30\,EX + 2\,EX^2 = 30(1.5)(3.5) + 2(1.5)^2(3.5)(4.5) = 228.4$

$E(L^2) = E[(30x + 2x^2)^2] = 78,378.9$

$V(L) = 26,233.75$.

b. $1 - \frac{1}{k^2} = 0.89$, so $k = 3.015$

$P(-259.8 < L < 716.6) \geq 0.89$

5.69 Let X_i be the time to completion of a given task, $i = 1, 2$. Let $Y = X_1 + X_2$ each X_i follows a gamma distribution with parameters $\alpha = 1$, $\beta = 10$ and Y follows a gamma dist. with parameters $\alpha = (1)(2)$, $\beta = 10$

a. $EY = (2)(10) = 20$

$V(Y) = (2)(10)^2 = 200$

b. Let A = average time to complete tasks

$E(A) = E(\frac{1}{2}Y) = 10$

$V(A) = V(\frac{1}{2}Y) = \frac{200}{4} = 50$