Chp 4

130.- $M(t) = (pe^t + q)^n$

$$m'(t) = npe^t(pe^t + q)^{n-1}$$

$$EX = m'(0) = np.$$

$$m''(t) = npe^t(pe^t + q)^{n-1} + (n)(n-1)p^2 e^{2t}(pe^t + q)^{n-2}$$

$$EX^2 = m''(0) = np + n(n-1)p^2.$$

$$V(x) = EX^2 - (EX)^2 = np + n(n-1)p^2 - (np)^2 = np(1-p).$$

131. $M(t) = e^{\lambda(e^t - 1)}$

$$m'(t) = e^t \lambda e^{\lambda(e^t - 1)}$$

$$EX = m'(0) = \lambda$$

$$m''(t) = (\lambda e^t)^2 e^{\lambda(e^t - 1)} + \lambda e^t e^{\lambda(e^t - 1)}$$

$$EX^2 = m''(0) = \lambda^2 + \lambda$$

$$V(x) = E(x^2) - (EX)^2 = \lambda^2 + \lambda - \lambda^2 = \underline{\lambda}$$

139.

States:

S_1: rain

S_2: Sunny

$$P = \begin{array}{c|cc} & S_1 & S_2 \\ \hline S_1 & 0.1 & 0.9 \\ S_2 & 0.3 & 0.7 \end{array}$$

We need to find π the stationary distribution.

We need to solve

$\pi = \pi P$ Subject to $\pi_1 + \pi_2 = 1$.

So,

$$[\pi_1 \quad \pi_2] = [\pi_1 \quad \pi_2]\begin{bmatrix} 0.1 & 0.9 \\ 0.3 & 0.7 \end{bmatrix} = [0.1\pi_1 + 0.3\pi_2 \quad 0.9\pi_1 + 0.7\pi_2]$$

hence

$\pi_1 = 0.1\pi_1 + 0.3\pi_2$ ... (1)

$\pi_2 = 0.9\pi_1 + 0.7\pi_2$ ... (2)

From (1)

$\pi_1 = 0.1\pi_1 + 0.3\pi_2$ add $0.2\pi_1$ to both sides

$1.2\pi_1 = 0.3\pi_1 + 0.3\pi_2$

$1.2\pi_1 = 0.3(\pi_1 + \pi_2)$, but $\pi_1 + \pi_2 = 1$, so

$\pi_1 = \frac{0.3}{1.2} = 1/4$.

and $\pi_2 = 3/4$, Solution $\boxed{\pi_2 = 3/4}$ or 75% of the days is Sunny.

140.

a)

$$P = \begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0.3 & 0.3 & 0.4 \\ 0.4 & 0.4 & 0.2 \\ 0.5 & 0.3 & 0.2 \end{bmatrix} \end{array}$$