4.66 $p=0.6$

a. $r=2$

$$P(X=x)=p(x)=\binom{x+r-1}{r-1}p^r(1-p)^x,\quad x=0,1,2,\ldots$$

So

$$P(X=x)=\binom{x+1}{1}(0.6)^2(0.4)^x,\quad x=0,1,2,\ldots$$

then

$$P(X\geq 3)=1-P(X\leq 2)=1-[P(X=0)+P(X=1)+P(X=2)]$$

$$=1-\left[\binom{1}{1}(0.6)^2(0.4)^0+\binom{2}{1}(0.6)^2(0.4)+\binom{3}{1}(0.6)^2(0.4)^2\right]$$

$$=1-[0.36+0.288+0.1728]$$

$$=1-[0.8208]=0.1792$$

b.

Analog to b.

4.67 $p = 0.1$

$P(\text{first two defective and then a good one}) = (0.10)^2(0.9)$

$= \underline{0.009}$

4.69 Let $X = $ # of trials to obtain r successes

"A success is a nondefective engine", $p = 0.9$

a. Using formula $p(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$, $x = r, r+1, r+2, \ldots$

from page 147.

$P(X=7) = \binom{6}{4}(0.9)^5(0.1)^2 = \frac{6!}{4!\,2!}(0.9)^5(0.1)^2 = (3)(5)(0.9)^5(0.1)^2 = \underline{0.09}$

b. $P(X=10) = \binom{9}{4}(0.9)^5(0.1)^5 = \frac{9!}{4!\,5!}(0.9)^5(0.1)^5 =$

$= \frac{9\times8\times7\times6}{4\times3\times2\times1}(0.9)^5(0.1)^5 = \frac{9\times2\times7\times2}{2\times1}(0.9)^5(0.1)^5 = \underline{0.00074}$

4.70

a. Let $X =$ # of failures before first success.

$$EX = \frac{q}{p} = \frac{0.1}{0.9} = \underline{0.111}$$

$$V(X) = \frac{q}{p^2} = \frac{(0.1)}{(0.9)^2} = 0.123$$

b. Let $X =$ # of failures before the third success.

X is negative binomial, $p = 0.9$, $r = 3$.

$$EX = (3)\frac{(0.1)}{(0.9)} = \underline{0.333}$$

$$V(x) = \frac{(3)(0.1)}{(0.9)^2} = \underline{0.37}$$

4.84

Let $X = \#$ of games the child plays.

X is geometric(p)

a. Total $= X + 4$

so

$$E[\text{Total}] = E[X+4] = EX + 4 = \frac{1-p}{p} + 4.$$

4.90

Let $X = \#$ of call on a given minute.

X is Poisson(λ) with $\lambda = 3$.

a. $$P(X=0) = \frac{3^0 e^{-3}}{0!} = e^{-3} = 0.05$$

b. $$P(X \geq 2) = 1 - P(X \leq 1) = 1 - [P(X=0) + P(X=1)]$$

$$= 1 - [0.05 + 3e^{-3}]$$

$$= 1 - [0.05 + 0.15] = \underline{0.80}.$$

c. X = # of Call in a 2 minute Period.

X is poisson (λ) with $\lambda = (2)(3) = 6$.

$$P(X \geq 2) = 1 - [P(X=0) + P(X=1)]$$

$$= 1 - \left[\frac{6^0 e^{-6}}{0!} + \frac{6 e^{-6}}{1}\right]$$

$$= 1 - [0.002 + 0.015] = \underline{0.983}.$$

4.91

Let X = # hurricanes during a Year.

X is poisson (λ) with $\lambda = 0.45$

a. $P(X \geq 1) = 1 - P(X=0) = 1 - e^{-0.45} = \underline{0.363}$

b. $P(X \leq 4) = \underline{0.999}$.

4.92

a. Assume X is poisson (λ) with $\lambda = 0.5$

b. $P(X \geq 1) = 1 - P(X=0) = 1 - e^{-0.5} = 0.4$

c. X is poisson (λ) with $\lambda = (0.5)(5) = 2.5$

$P(X \geq 1) = 1 - P(X=0) = 1 - e^{-2.5} = 1 - 0.082 = 0.918$

4.94 X is poisson (λ) with $\lambda = (0.5)(10) = \underline{5}$.

(X = # of flaws in 10 m).

Cost = $8X$

So

$E[\text{cost}] = E[8X] = 8E[X] = (8)(5) = \underline{40}$.

$V(\text{cost}) = V(8X) = (8)^2 V(X) = (64)(5) = 320$,

So $SD(\text{cost}) = \sqrt{320} \approx 17.9$

4.98 Let $X =$ # of particles in an hour.

X is Poisson (λ) with $\lambda = 4$

a. $P(X \geq 6) = 1 - P(X \leq 5) = 1 - [0.78 \quad] = 0.21$

b. $P(X \leq 3) = 0.43$

c. $P(X=0) = 0.018$

4.109 Let $X =$ # of white balls selected.

a. $P(X=1) = \dfrac{\binom{5}{1}\binom{9-5}{2-1}}{\binom{9}{2}} = \dfrac{\binom{5}{1}\binom{4}{1}}{\binom{9}{2}}$

b. $P(X \geq 1) = 1 - P(X=0) = 1 - \left[\dfrac{\binom{5}{0}\binom{4}{2}}{\binom{9}{2}}\right]$

c. $P(X=2 \mid X \geq 1) = \dfrac{P(X \geq 1 \cap X=2)}{P(X \geq 1)}$

$$= \frac{P(X=2)}{P(X \geq 1)} = \frac{\dfrac{\binom{5}{2}\binom{4}{0}}{\binom{9}{2}}}{P(X \geq 1)}$$

d. $P(2^{nd}$ ball is white$) = P(X=2) + P($first red and 2^{nd} white$)$

A.112

a. Let $X = \#$ white Jurors

$$P(X=0) = \frac{\binom{12}{0}\binom{15}{12}}{\binom{27}{12}}$$

b. Let $X = \#$ black Jurors.

$$P(X=9) = \frac{\binom{9}{9}\binom{18}{3}}{\binom{27}{12}}$$

c. Let $X = \#$ hispanic and asians on the Jury

$$P(X=0) = \frac{\binom{6}{0}\binom{21}{12}}{\binom{27}{12}}$$

7.119

a.

X	0	1	2
$P(x)$	$\frac{\binom{2}{0}\binom{8}{3}}{\binom{10}{3}} = 0.47$	$\frac{\binom{2}{1}\binom{8}{2}}{\binom{10}{3}} = 0.47$	$\frac{\binom{2}{2}\binom{8}{1}}{\binom{10}{3}} = 0.07$

b. Analog to (a)

X	0	1	2	3
$P(X)$	$\frac{\binom{4}{0}\binom{6}{3}}{\binom{10}{3}}$			