4.57

$n=15$, $P=0.06$ $Cost=550$

Let $X=\#$ of defective ranges out of the total

$Gain = n(cost) - 1100X$

$E[Gain] = E[(15)(550) - 1100X]$

$= 8250 - 1100EX$, $EX = (15)(0.06) = 0.9$

$= 8250 - (1100)(0.9) = \boxed{7260}$

4.58 X is binomial(n,p)

$P(\text{reject}) = P(X \geq 1) = 1 - P(X=0) = 1 - \left[\binom{n}{0} p^0 (1-p)^n\right] =$

$= 1 - (1-p)^n = 0.95$

, then solving for n

$(1-p)^n = 0.05$

$\Rightarrow n\ln(1-p) = \ln(0.05)$

$\Rightarrow n = \dfrac{\ln(0.05)}{\ln(1-p)}$

thus

4.45 Let $X = \#$ of cartons underfilled.

a. X is binomial $(50, 0.05)$, where a success is carton underfilled. Then

$$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= \binom{50}{0}(0.05)^0(0.95)^{50} + \binom{50}{1}(0.05)^1(0.95)^{49}$$

$$+ \binom{50}{2}(0.05)^2(0.95)^{48} = 0.5405$$

b. X is binomial $(50, 0.1)$

$$P(X \le 2) = 0.1117$$

46.

Let X = # of insects that survive.

X is binomial $(40, 0.50)$

a. $P(X=20) = \binom{40}{20}(0.50)^{20}(0.50)^{20} = 0.1254$

b. $P(X \leq 15) = \sum_{x=0}^{15} \binom{40}{x}(0.50)^{x}(0.50)^{20-x} = 0.0769$

c. $P(X \geq 20) = 1 - P(X \leq 19) = 1 - 0.437 = 0.562$

d. We need independent trials to use the binomial distributio, so the insects must be treated separately.

e. $EX = np = (40)(0.5) = 20$

f. $V(x) = np(1-p) = 40(0.5)^2 = 10$

So $SD(x) = \sqrt{10} = 3.16$

4.18

a. Let $X =$ # of businesses with a female owner only

$$P = \frac{6.5}{6.5+13.2+2.7} = 0.29$$

X is binomial$(4, 0.29)$

So $P(X=4) = \binom{4}{4}(0.29)^4(0.71)^0 = \boxed{0.007}$

b. Let $X =$ # of businesses owned by a male

$$P = \frac{13.2+2.7}{6.5+13.2+2.7} = 0.71$$

X is binomial $(4, 0.71)$

$P(X=1) = \binom{4}{1}(0.71)^1(0.29)^3 = \boxed{0.07}$

c. $X =$ # of businesses jointly owned

$$P = \frac{2.7}{6.5+13.2+2.7} = 0.12$$

X is binomial$(4, 0.12)$

$P(X=0) = \binom{4}{0}(0.12)^0(0.88)^4 = \boxed{0.60}$

1.49

a. The inspections are independent events

b. A success is to dectect a wing crack,

$P = (0.9)(0.8)(0.5) = 0.36$

Let X = # of Planes win crack.

X is binomial $(3, 0.36)$

$$P(X \geq 1) = 1 - P(X=0) = 1 - \binom{3}{0}(0.36)^0(0.64)^3$$

$$= 1 - 0.2621 = \underline{0.7379}$$

4.52 Let $X = \#$ of people contracting mumps

$p = 0.20$

a. X is binomial $(2, 0.20)$

$$P(X \geq 1) = 1 - P(X=0) = 1 - \binom{2}{0}(0.20)^0(0.80)^2$$

$$= 1 - 0.64 = \boxed{0.36}$$

b. X is binomial $(4, 0.20)$

$$P(X \geq 1) = 1 - P(X=0) = 1 - \binom{4}{0}(0.20)^0(0.80)^4$$

$$= 1 - 0.4096 = \boxed{0.5904}$$

4.55

X is binomial $(4, 0.12)$

So

$$E[C] = E[2X^2 + X + 3]$$

$$= 2E[X^2] + E[X] + 3$$

note that $V(X) = E(X^2) - (E(X))^2$, so

$$E(X^2) = V(X) + (E(X))^2$$

then

$$E[C] = 2\left(V(X) + (E(X))^2\right) + E(X) + 3$$

and $EX = np = (4)(0.12) = 0.48$

$V(X) = np(1-p) = 4(0.12)(0.88) = 0.42$

So

$$E[C] = 2\left(0.42 + (0.48)^2\right) + (0.48) + 3$$

$$= \boxed{4.78}$$

a. $p = 0.10$

$$n = \frac{\ln(0.05)}{\ln(0.9)} = 28.43, \text{ let } n = 29.$$

b. $p = 0.05$

$$n = \frac{\ln(0.05)}{\ln(0.95)} = 58.40, \quad n = 59$$

~~#103~~

4.65 $X = \#$ of failures before the first success

$p = 0.1$

a. $P(X \geq 2) = (0.9)(0.9) = 0.81$

or $P(X \geq 2) = 1 - [P(X=0) + P(X=1)] = 1 - [0.1 + 0.09] = 1 - 0.19 = \underline{0.81}$

b. $$P(X \geq 4 \mid X \geq 2) = \frac{P((X \geq 4) \cap (X \geq 2))}{P(X \geq 2)}$$

$$= \frac{P(X \geq 4)}{P(X \geq 2)} = \frac{(0.9)^4}{(0.9)^2} = (0.9)^2 = P(X \geq 2) = \underline{0.81}$$