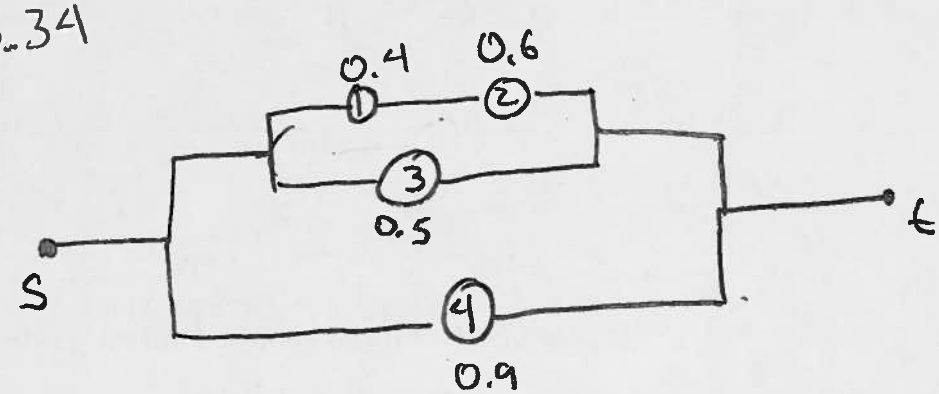
3.34



Let F = event that the current will flow.

then

$$P(F) = 1 - P(\bar{F}),$$

So

$P(\bar{F}) = P($ 4 and 3 are open and [(1 open and 2 closed) or (1 closed and 2 open)] $)$

$$= (0.1)(0.5)\left[(0.6)(0.6) + (0.6)(0.4)\right]$$

$$= (0.05)\left[0.36 + 0.24\right] = \underline{0.03}$$

$$P(F) = 1 - 0.03 = \boxed{0.97}$$

3.36 By Contradiction

Suppose A and B are mutually exclusive,

that is $AB = \emptyset$

then $P(AB) = P(\emptyset) = 0$

but we are given that A and B are

independent, so $P(AB) = P(A)P(B) > 0$, since

$P(A) > 0$ and $P(B) > 0$, contradiction!

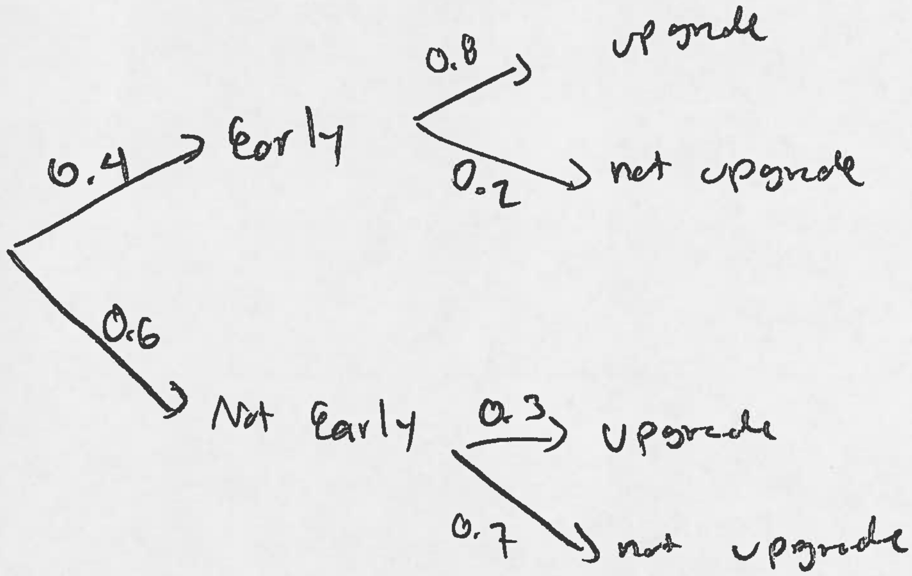
It's impossible for A and B be mutually exclusive

or

$P(AB) = P(A)P(B) > 0$, since $P(A) > 0$, $P(B) > 0$

So $P(AB) \neq 0$, $AB \neq \emptyset$.

3.43



Using the Total Probability law

$P(\text{upgrade}) = P(\text{upgrade} \mid \text{Early})P(\text{early}) + P(\text{upgrade} \mid \text{not early})P(\text{not early})$

$$= (0.8)(0.4) + (0.3)(0.6) = \underline{0.5}$$

3.44

we have

Sensivity = P(positive test | have disease) = 0.95

Specificity = P(negative test | no disease) = 0.95

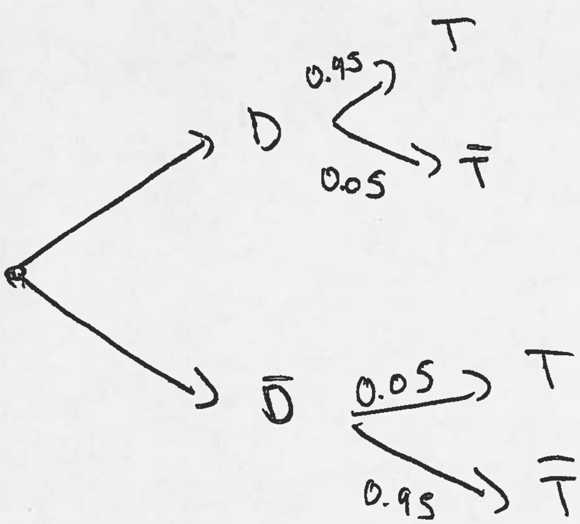
Let D = "having disease" and T = "positive test".

$P(D) = 0.01$

So, we are looking for P(have disease | test positive) =

$$P(D|T) = \frac{P(T|D)P(D)}{P(T)} = \frac{P(T|D)P(D)}{P(T|D)P(D) + P(T|\bar{D})P(\bar{D})}$$

$$= \frac{(0.95)(0.01)}{(0.95)(0.01) + (0.05)(0.99)} = \boxed{0.16}$$



3.47 Assume all randomly selected people have an equal probability of having HIV.

P(Person with unknown status will have the disease)

$= P(\text{disease})P(\text{unaware}|\text{disease}) = (0.0038)(0.248) = \underline{0.000942}$

3.48

a. $P(\text{Pacific} \cap \text{soda}) = P(\text{Pacific})P(\text{Soda}|\text{Pacific}) = (0.04)(0.71) = \underline{0.0284}$

b. $P(\text{PoP}) = P(\text{PoP}|\text{Pacific})P(\text{Pacific}) + P(\text{PoP}|\text{Rocky mountains})P(\text{Rocky m})$

$+ \ldots + P(\text{PoP}|\text{Southeast})P(\text{Southeast}) =$

$(0.15)(0.04) + (0.61)(0.10) + (0.12)(0.06) + (0.70)(0.18)$

$+ (0.30)(0.28) + (0.18)(0.34) = \underline{0.35}$

c. $P(\text{Southeast}|\text{Coke}) = \dfrac{P(\text{Southeast} \cap \text{Coke})}{P(\text{Coke})} = \dfrac{P(\text{Southeast})P(\text{Coke}|\text{Southeast})}{P(\text{Coke})}$

$= \dfrac{(0.06)(0.39)}{P(\text{Coke})}$

$P(\text{Coke}) = P(\text{Coke}|\text{Pacific})P(\text{Pacific}) + P(\text{Coke}|\text{RM})P(\text{RM}) + \ldots + P(\text{Coke}|\text{Southeast})P(\text{South}$

3.64

P(at least one gets the flu) = 1 − P(none)

P(none) = P(not the vaccinated ∩ not the unvaccinated)

= P(not the vaccinated) P(not the unvaccinated)

= (0.8)(0.1)

thus

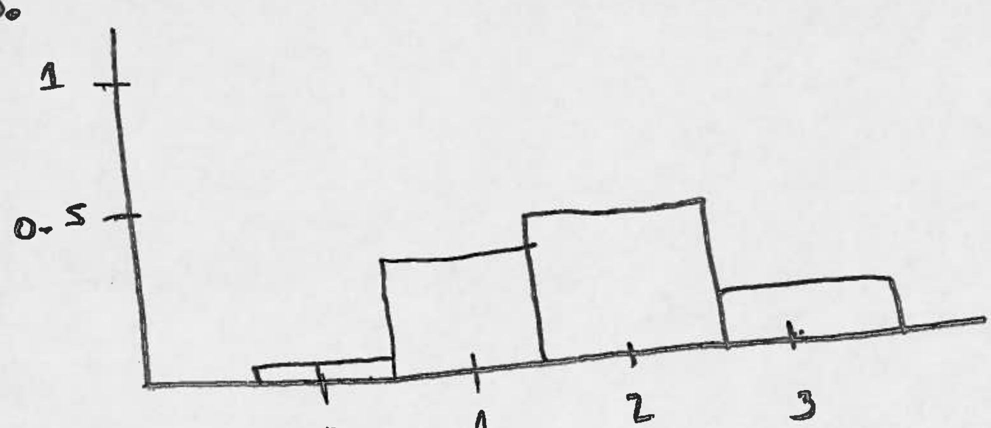
P(at least one) = 1 − (0.8)(0.1) = 0.92

4.2 Let X = # of females

a.

X	0	1	2	3
$p(x)$	$\frac{\binom{7}{0}\binom{5}{3}}{\binom{12}{3}} = 0.045$	$\frac{\binom{7}{1}\binom{5}{2}}{\binom{12}{3}} = 0.32$	$\frac{\binom{7}{2}\binom{5}{1}}{\binom{12}{3}} = 0.47$	$\frac{\binom{7}{3}\binom{5}{0}}{\binom{12}{3}} = 0.165$

b.



c.

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.042, & 0 \leq x < 1 \\ 0.365, & 1 \leq x < 2 \\ 0.835, & 2 \leq x < 3 \\ 1, & x \geq 3 \end{cases}$$

d. Similar to what we did in class,

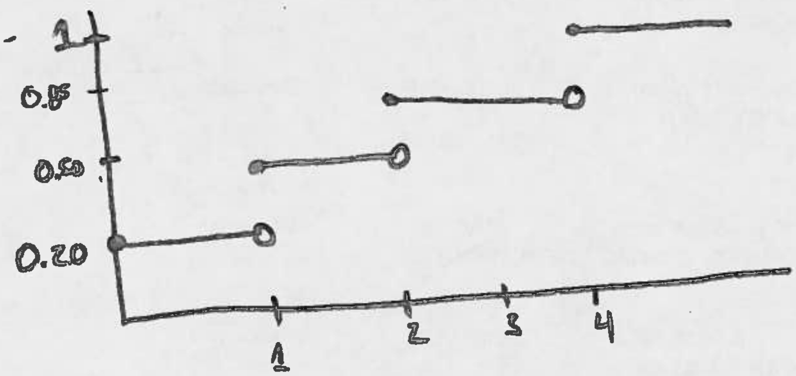
4.8 Let X = # of defective chips among the two inspected

X	0	1	2
$P(x)$	$\frac{\binom{2}{0}\binom{2}{2}}{\binom{4}{2}} = \frac{1}{6}$	$\frac{\binom{2}{1}\binom{2}{1}}{\binom{4}{2}} = \frac{2}{3}$	$\frac{\binom{2}{2}\binom{2}{0}}{\binom{4}{2}} = \frac{1}{6}$

b. $P(X \le 1) = P(X=0) + P(X=1) = \frac{5}{6}$.

4.12

a.



b. $P(X=0) = 0.20$

$P(X=1) = P(X \le 1) - P(X \le 0) = 0.50 - 0.20 = 0.30$

$P(X=2 \text{ or } X=3) = P(X \le 3) - P(X \le 1) = 0.85 - 0.50 = 0.35$

$P(X=4) = P(X \le 4) - P(X \le 3) = 1 - 0.85 = 0.15$

c.

4.33

Let X = # of items to be purchased on a given day.

Y = # of items stocked

then

$$E X = (2)(0.2) + (3)(0.3) + (4)(0.5) = \underline{2.6}.$$

Since we are expecting to sell 2.6 items per day, we need to stock 3 items,

so $\underline{Y = 3}$.

4.42 From the dedinition

$$E X = \sum_{x=0}^{\infty} x P(X=x)$$

So

$$\sum_{x=0}^{\infty} x P(X=x)$$

$$\begin{aligned}
&\parallel \\
&0 \cdot P(X=0) &&= 0 \\
&+ \\
&1 \cdot P(X=1) &&= P(X=1) \\
&+ \\
&2 \cdot P(X=2) &&= P(X=2) + P(X=2) \\
&+ \\
&3 \cdot P(X=3) &&= P(X=3) + P(X=3) + P(X=3) \\
&+ \\
&4 \cdot P(X=4) &&= P(X=4) + P(X=4) + P(X=4) + P(X=4) \\
&+ &&= P(X=5) + \cdots + P(X=5) \\
&\vdots
\end{aligned}$$

So

$$E X = \sum_{x=1}^{\infty} x P(X=x) = \sum_{x=1}^{\infty} \sum_{n=0}^{x-1} P(X=x) = \sum_{n=0}^{\infty} \sum_{x=n+1}^{\infty} P(X=x) = \underline{\sum_{n=0}^{\infty} P(X>n)}.$$

4.17 Let B_i = Gain for box i, for $i=1,2$.

Let X = number on ticket that was drawn

Then

$$B_i = X - 1$$

a.

$$EB_1 = E(X-1) = (0-1)\tfrac{1}{3} + (1-1)(\tfrac{1}{3}) + (2-1)(\tfrac{1}{3}) = 0$$

$$EB_1^2 = E(X-1)^2 = (0-1)^2\tfrac{1}{3} + (1-1)^2(\tfrac{1}{3}) + (2-1)^2(\tfrac{1}{3}) = \tfrac{2}{3}.$$

thus

$$V(B_1) = EB_1^2 - (EB_1)^2 = \tfrac{2}{3}.$$

b. Similar to a.

$$EB_2 = 0$$
$$EB_2^2 = \tfrac{12}{5}$$
$$V(B_2) = \tfrac{12}{5}$$

c. It depends on my mood, with box 2 we can win up to 3 dollars but the variance is higher than the box 1. and in box 1 we can win up to 1 dollar.

4.26

a. $P(win) = \frac{1}{\binom{50}{9}}$

b. $E[win] = \frac{1}{\binom{50}{9}} \cdot 5000\,000 = 0.001995$

$V(win) = E[win^2] - E[win]^2$

$= \left(\frac{1}{\binom{50}{9}}\right)(5000\,000)^2 - (0.001995)^2$

$= \underline{9974.99}$

c.

$E[win] = \left(\frac{1}{\binom{50}{9}}\right)(5000\,000 - 0.5) + (-0.5)\left(1 - \frac{1}{\binom{50}{9}}\right) = -0.98$

$V[win] = E[win^2] - E[win]^2$

$= \left(\frac{1}{\binom{50}{9}}\right)(5000\,000 - 0.5)^2 + (-0.5)^2\left(1 - \frac{1}{\binom{50}{9}}\right) - (-0.98)^2 = \underline{9974.29}$