3.6 Let P = "event that Elise passes the test"

Let S = "event that Elise studies"

then

$$P(P|S) = \frac{P(P\cap S)}{P(S)} = \frac{0.8}{0.9} = 0.89$$

3.8 Let R = "Jessica remembers and stops by the store"

B = "Buys cat food"

$$P(R\cap B) = P(R)\,P(B|R) = (0.6)(0.5) = 0.3$$

3.10

Let the sample $S = \{(1,1),(1,2),(1,3)$
$(2,1),(2,2),(2,3)$
$(3,1),(3,2),(3,3)\}$,

where $(1,2)$ the first entry corresponds to the paper contract and the second entry corresponds to the DVD+RWs contract. 1 corresponds to firm 1, 2 - firm 2 and 3 - firm 3.

a. $P(1,1) = \frac{1}{9}$

b. P(firm 1 received a contract, given that both contracts do not go the same firm) $= \frac{4}{6} = \frac{2}{3}$.

c. P(firm 1 receives the contract paper, given it does not receive the contract for the DVD+RWs)

$= \frac{2}{6} = \frac{1}{3}$.

d. We assume that each firm has equal probability of receiving each contract.

3.11

There are $\binom{20}{2}$ Possible Combination of Size 2 out of 20 different items.

a. P(neither is detective) = $\dfrac{\text{\# Combinations of Size 2 with no defec}}{\text{\# Possible Combinations}}$

$$= \frac{\binom{4}{0}\binom{16}{2}}{\binom{20}{2}} = \frac{12}{19} = \underline{0.632}$$

b. P(at least one of the two is defective) =

P(one is defective or two are defective) = $\dfrac{\binom{4}{1}\binom{16}{1} + \binom{4}{2}\binom{16}{0}}{\binom{20}{2}}$

$= 0.368$

or

$= 1 - P(\text{no defective}) = 1 - 0.632 = \underline{0.368}$

c. P(neither is defective, given that at least one is defective) = $\dfrac{\binom{4}{0}\binom{16}{2}}{\binom{20}{2}} \Big/ 1 - \dfrac{\binom{4}{2}\binom{16}{0}}{\binom{20}{2}} = 0.652$

3.14

$$P(ABC) = P(AB)\,P(C \mid AB)$$

$$= P(A)\,P(B \mid A)\,P(C \mid AB)$$

3.20

Next

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3.20

We have 5 Cards and we Know two of them are Jacks. There a two ways to complete the full house:

1. The two Jacks and three of a Kind of another Card type.

2. the two Jacks, another Jack and two of a Kind of another Card type.

Counts for ① $= 10\binom{4}{3} + 2\binom{3}{3}$, Since there are 10 Possible Card types for the player to be dealt three of a Kind (A, 3, 4, 5, 6, 7, 8, 9, Q, K) and the player needs 3 of those 4. and for the Card types 2 and 10 there are only 3 of those left and the player would need all three.

Counts for ② $= \binom{2}{1}\left(10\binom{4}{2} + 2\binom{3}{2}\right)$

Choose a Pair from 3

For Card types 2 and 10,

two options to get another Jack

Choose a Pair

10 different Kinds

hence,

$$P(\text{full house}) = \frac{10\binom{4}{3} + 2\binom{3}{3} + \binom{2}{1}\left(10\binom{4}{2} + 2\binom{3}{2}\right)}{\binom{48}{3}}$$

$$= 0.01.$$

3.21 $\Rightarrow$

Let $P(B|A) > P(B)$, we need to show

$P(A|B) > P(A)$.

So, $P(B|A) > P(B)$

$P(B|A)P(A) > P(B)P(A) \quad (P(AB) = P(A)P(B|A))$

then $P(AB) > P(B)P(A)$

then $\dfrac{P(AB)}{P(B)} > P(A) \quad \left(P(A|B) = \dfrac{P(AB)}{P(B)}\right)$

thus $P(A|B) > P(A)$.

$\Leftarrow$) Analog to the previous one.

3.23

Partially solved in class.

3.27

a. $P(\text{Pass}|\text{male}) = \frac{24}{40} = 0.6$

b. $P(\text{male}|\text{Pass}) = \frac{24}{60} = 0.4$

c. $P(\text{Pass}|\text{male}) = 0.6$ and

$P(\text{Pass}) = \frac{60}{100} = 0.6$

So

$P(\text{pass}|\text{male}) = P(\text{pass})$, thus they are independent.

3. 28

Let X= "event of rolling an 8"

Y= "event of rolling doubles"

Let S be the sample space, $|S| = 6 \cdot 6 = \underline{36}$

X and Y are independent if $P(XY) = P(X) \cdot P(Y)$

$P(X) = \frac{5}{36} = 0.14$, $X = \{(4,4), (6,2) (5,3), (2,6), (3,5)\}$

$P(Y) = \frac{6}{36} = 0.167$, $Y = \{(1,1), (2,2), (3,3), (4,4) (5,5), (6,6)\}$

$P(XY) = \frac{1}{36} = 0.027$, $XY = \{(4,4)\}$

and

$P(XY) = 0.027 \neq (0.14)(0.167) = 0.02338$

3.29

$P(\text{Jack}) = \frac{4}{52} = \frac{1}{13}$

$P(\text{Heart}) = \frac{13}{52} = \frac{1}{4}$

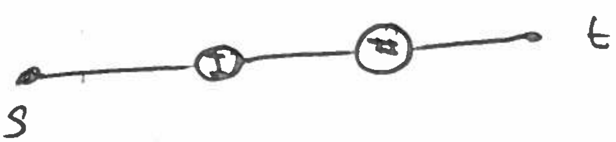
$P(\text{Jack and Heart}) = \frac{1}{52}$

and

$P(\text{Jack and Hert}) = \frac{1}{52} = \frac{1}{13} \cdot \frac{1}{4} = P(\text{Jack}) P(\text{Heart})$

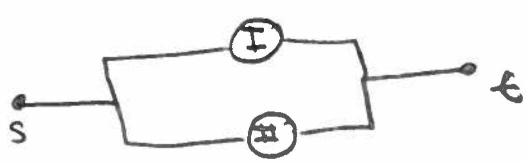
So, they are independent.

3.32



$P(\text{current}) = P(\text{close})P(\text{close}) = (0.9)(0.9) = 0.81$

3.33



$P(\text{electricity flows}) = P(\text{I closes or II closes})$

$= P(\text{I closes}) + P(\text{II closes}) - P(\text{both})$

$= 0.9 + 0.9 - 0.9(0.9) = \underline{0.99}$.