2.10

a. $S = \{$ (mary, Jim) (Jim, Don) (Sue, Don) (Don, nancy)
(mary, Don) (Jim, Sue) (Sue, Nancy)
(mary, Sue) (Jim, Nancy)
(mary, Nancy) $\}$

b. A = "at least one man is selected" = { (mary, Jim), (mary, Don),
(Jim, Don), (Jim, Sue), (Jim, Nancy), (Sue, Don), (Don, nancy) }

$|A| = 7$

c. B = "exactly one male is selected" = { (mary, Jim), (mary, Don)
(Jim, Sue), (Jim, Nancy), (Sue, Don), (Don, Nancy) }

$|B| = 6$

d. not graded

e.

$\bar{A} = \{$ (mary, sue), (mary, nancy), (sue, nancy) $\}$

$AB = B$

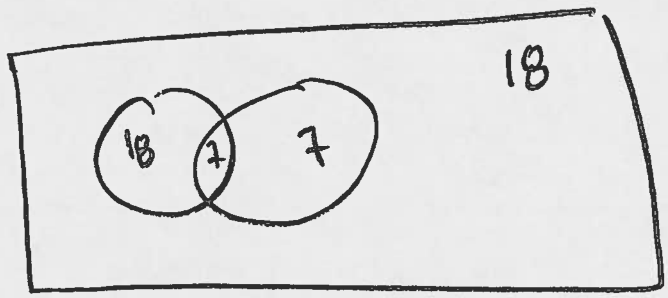
$A \cup B = A$

$\overline{AB} = \bar{B} = \{$ (mary, sue), (mary, nancy), (Jim, Don),
(sue, nancy) $\}$.

2.8

D = event the household has a dog

C = event the house hold has a cat



a. $D \cap \bar{C} = 18$

b. $C = 14$

c. $C \cap \bar{D} = 7$

d. $(D \cap \bar{C}) \cup (C \cap \bar{D}) = 18 + 7 = \underline{25}$

2.14

Similar to

2.8

Use

$$TV \cap CD = (TV \cap CD \cap CPU) \cup (TV \cap CD \cap \overline{CPU})$$

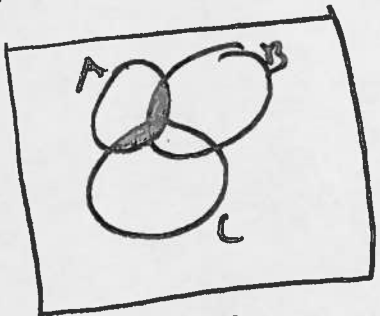
15) Distributive laws

$A(B \cup C) = AB \cup AC$

$A \cup (BC) = (A \cup B) \cap (A \cup C)$

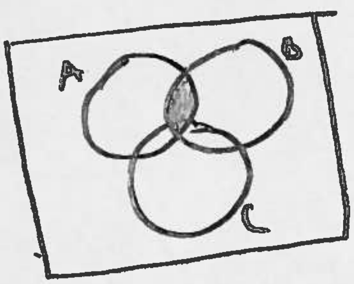
(1)

Not Strict

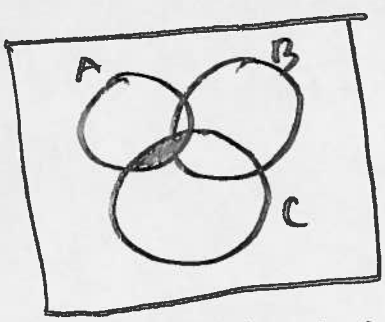


$A(B \cup C)$ is shaded

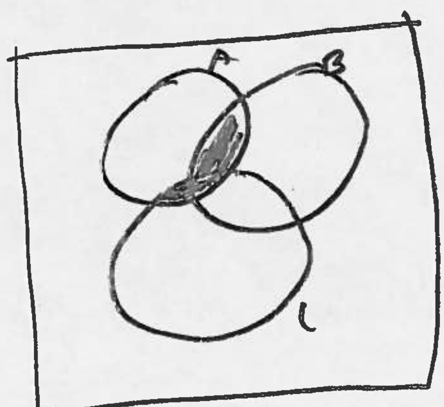
ow



AB is shaded



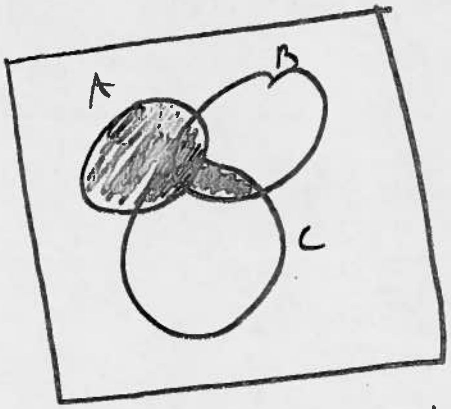
AC is shaded



$AB \cup AC$ is shaded

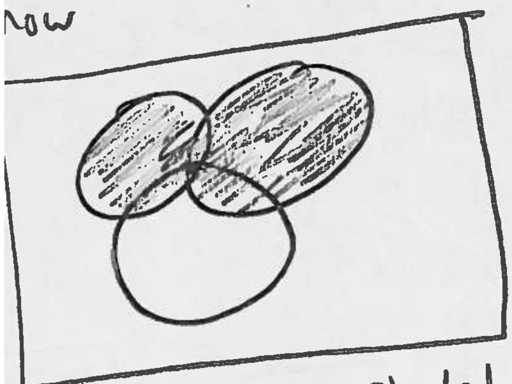
which is the same shaded are as $A(B \cup C)$.

(2)

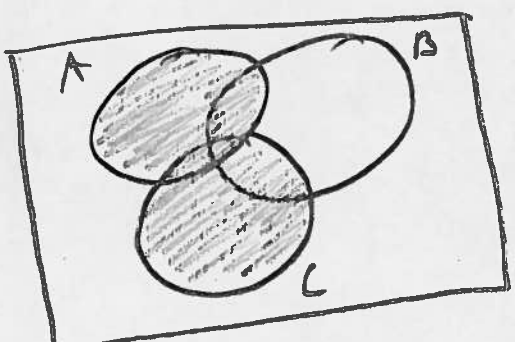


$A \cup (BC)$ is shaded

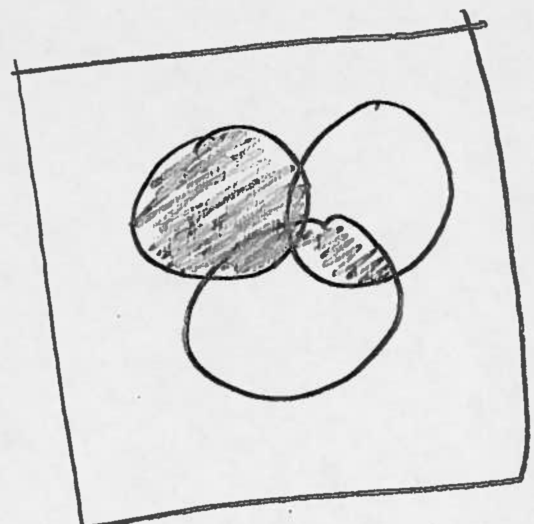
now



$A \cup B$ is shaded



$A \cup C$ is shaded



$(A \cup B) \cap (A \cup C)$ is shaded

which is the same shaded area as $A \cup (BC)$.

2.18

a. Let $A =$ "both males are selected".

$A = \{(\text{Jim}, \text{Don})\}$ then

$$P(A) = \frac{|A|}{|S|} = \frac{1}{10}.$$

b. Let $B =$ "at least one male is selected".

Using Prob. 2.10 b. $P(B) = \frac{|B|}{|S|} = \frac{7}{10}$.

c. Let $C =$ "at least one female is selected"

Note that $\bar{C} =$ "no female is selected" = "both males are selected" $= A$.

then

$$P(C) = 1 - P(\bar{C}) = 1 - P(A) = 1 - \frac{1}{10} = \frac{9}{10}.$$

2.20

Let A denote the event that a potential customer buys from outlet 1.

Let B denote the event that a potential customer buys from outlet 2.

a. "The customer buys from outlet 1" $= A$.

b. "The customer does not buy from outlet 2" $= \overline{B}$

c. "the customer does not buy from outlet 1 or does not buy from outlet 2" $= \bar{A} \cup \bar{B} = \overline{A \cap B}$.

d. "The customer does not buy from outlet 1 and does not buy from outlet 2 $= \bar{A} \cap \bar{B} = \overline{(A \cup B)}$

2.27

Proof: A can be expressed as $(A \cap B) \cup (A \cap \bar{B})$,

which are mutually exclusive, then

$$P(A) = P(A \cap B \cup A \cap \bar{B}) = P(A \cap B) + P(A \cap \bar{B})$$

but $P(A \cap \bar{B}) \geq 0$, by axiom 1, then

$$P(A) \geq P(A \cap B). \quad \blacksquare$$

2.2B

Claim: $P(AB) \geq P(A) + P(B) - 1$

<u>Proof:</u> $A \cup B = A \cup \bar{A}B$, A and $\bar{A}B$ are disjoint.

now, $S = (A \cup B) \cup (\overline{A \cup B})$, so $1 = P(S) = P(A \cup B \cup \overline{A \cup B})$, so

$1 = P(A \cup B) + P(\overline{A \cup B})$, then

$1 \geq P(A \cup B) = P(A \cup \bar{A}B) = P(A) + P(\bar{A}B)$, by axiom (3).

now $B = AB \cup \bar{A}B$, AB and $\bar{A}B$ are disjoint

then $P(B) = P(AB) + P(\bar{A}B)$ by axiom (3)

thus

$1 \geq P(A) + P(\bar{A}B) = P(A) + P(B) - P(AB)$

it follows that $P(AB) \geq P(A) + P(B) - 1$. $\blacksquare$